

Entanglement dynamics of two-bipartite system under the influence of dissipative environments

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Abstract

An experimental scheme is suggested that permits a direct measure of entanglement of two-qubit cavity system. It is articulated on the cavity-QED technology utilizing atoms as flying qubits. With this scheme we generate two different measures of entanglement namely logarithmic negativity and concurrence. The phenomenon of sudden death entanglement (ESD) in a bipartite system subjected to dissipative environment will be examined.

Keywords: Quantum entanglement, entanglement dynamics, bipartite entangled state, concurrence, logarithmic negativity, filed damping, dissipative environments.

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1. Introduction

Entanglement is an essential feature of quantum mechanics that permits basic peculiarity between classical and quantum physics. It is at the heart of many applications of quantum information science, including quantum teleportation [1–4], quantum dense coding [5], quantum cryptography [6], and quantum computing [7]. Entanglement can reveal the nature of a nonlocal correlation between quantum systems that have no classical interpretation. Recently, the use of the cavity QED in quantum information processing becomes more interesting [8–12], and the entanglement generation and nonlocality test of two cavity fields have been explored [13–17]. Real quantum

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system are unavoidably subjected to their environments, and these reciprocal interactions often result in the dissipative evolution of quantum coherence and loss of useful entanglement. Decoherence may guide to both local and global dynamics, which may incite the eventual deterioration of entanglement [15]. Particularly, Yu and Eberly have shown that the entanglement of bipartite systems can decay to zero abruptly, in a finite time which depends upon the initial preparation of the atoms, a phenomenon termed entanglement sudden death (ESD)[18] and was recently observed in two sophisticatedly designed experiments with photonic qubits [19] and atomic band [20]. Furthermore, it has also been observed in cavity QED and trapped ion systems [21]. On the other hand, the phenomenon ESD has provoked many theoretical investigations in other bipartite systems involving pairs of atomic, photonic, and spin qubits [22–25], multipartite systems [26] and spin chains [27–29]. In addition, ESD has also been explored for different environments [16, 18, 30, 31]. However, numerous investigations on ESD in a variety of systems have been done so far, the question of ESD in interacting qubits remains yet open [32].

On the other hand, several methods [33–37] have been proposed for detecting and measuring entanglement without a full reconstruction of the state. These methods, although much simpler than the full state reconstruction, are not completely free of experimental difficulties, as they require either controlled unitary operations or some prior knowledge about the quantum state in question, or they can detect entanglement but not measure its amount. Furthermore, for two qubits the defining measure of entanglement is concurrence [38]. It is a good measure of entanglement in every sense and direct measure of the concurrence of two-photon pure entangled state was confirmed experimentally using linear optical means [39]. Nevertheless, it exists another computable measure of the entanglement called negativity [40], and thereby fill an important gap in the study of entanglement. It can be regarded as a quantitative version of Peres’ criterion for separability [41]. For a mixed quantum states, the two measures are different. We will show some analytic relations between the two previous measures of entanglement for the proposed system.

The aim of this paper is two-fold. First, we propose an efficient scheme for quantum teleportation to generate entangled number states of two-bipartite system under the influence of dissipative environments. Second, we present two different computable measures of entanglement namely, the logarithmic negativity and the concurrence and we compare their amounts and agree-

ments for different reservoirs. Thus we investigate the problem of ESD for a this proposed scheme.

The format of this paper is as follows. In section 2 we investigate the model for two-bipartite system with a simple dissipative reservoir and formulate their dynamical evolution by solving the master equation of motion. In section 3 we present the theory of two different measure of entanglement of two-bipartite system subjected to dissipative environments (the logarithmic negativity and the concurrence). In section 4 we study the entanglement dynamics of two-bipartite system in vacuum and thermal reservoirs. Then we compare the amounts of the two computable measures of entanglement for different initial entangled states. We find that for thermal reservoirs the ESD always exists but for the vacuum reservoirs the ESD can be exhibit with some entangled states. Finally in section 5 we conclude with a general remarks and future outlook.

2. Model

Recently, Zubairy et al [43] have proposed a new scheme in their investigation of the quantum disentanglement eraser. In this simple scheme , the concurrence can be directly measured from the visibility for an explicit class of entangle states. We propose here the same scheme but with some modifications. A two-level atom with the upper level $|e\rangle$ and the lower level $|g\rangle$ passes consecutively through cavity A, a field damping region and a cavity B as shown in figure A.1. The atom is initially prepared in the excited state $|e\rangle$ and the decay of the radiation field inside a cavity may be described by a model in which the mode of the field of interest is coupled to a whole set of reservoir modes. We assume that initially the two cavities are in vacuum state $|0\rangle$ and the atom always leaves the setup in the ground state $|g\rangle$.

In the interaction picture and the rotating-wave approximation, the Hamiltonian is simply

$$\begin{aligned}
H = & \hbar \sum_j \left[g_j^{(A)} b_j^{(A)\dagger} a_A e^{-i(\nu-\nu_j)t} + g_j^{*(A)} a_A^\dagger b_j^{(A)} e^{i(\nu-\nu_j)t} \right] \\
& + \hbar \sum_j \left[g_j^{(B)} b_j^{(B)\dagger} a_B e^{-i(\nu-\nu_j)t} + g_j^{*(B)} a_B^\dagger b_j^{(B)} e^{i(\nu-\nu_j)t} \right] \quad (1)
\end{aligned}$$

where $a_{A(B)}$ and $a_{A(B)}^\dagger$ are the destruction and creation operators of the mode of the electromagnetic field of frequency ν . $b_j^{A(B)}$ and $b_j^{\dagger A(B)}$ are the modes

of cavity A(B) of frequency ν_j which damp the field and $g_j^{(i)}$ is the coupling constant of the interaction between the electromagnetic field and the cavity. From the general analysis of system-reservoir interactions, with the Hamiltonian (1), we can obtain directly the master equation for the reduced density matrix for the field in the cavities as [42]

$$\begin{aligned}\dot{\rho}(t) = & - \sum_{i=A,B} \frac{\gamma^{(i)}}{2} (\bar{n}_i + 1) \left[a_i^\dagger a_i \rho(t) - 2a_i \rho(t) a_i^\dagger + \rho(t) a_i^\dagger a_i \right] \\ & - \sum_{i=A,B} \frac{\gamma^{(i)}}{2} \bar{n}_i \left[a_i a_i^\dagger \rho(t) - 2a_i^\dagger \rho(t) a_i + \rho(t) a_i a_i^\dagger \right]\end{aligned}\quad (2)$$

where $\gamma^{(i)}$ is the decay rate in the cavity, and $\bar{n}_i (i = A, B)$ are the average number of quanta at frequency ν in the thermal reservoir which surrounds the cavities A and B. If the reservoirs are at zero temperature, $\bar{n}_i = 0$, and the remaining terms are due to vacuum fluctuations.

In the general case, we consider the field states in Fock basis in two identical high-Q cavities A and B that represent a bipartite system containing the entangle field as

$$|\Psi\rangle_{AB}(0) = a_1|0\rangle_A|0\rangle_B + a_2|0\rangle_A|1\rangle_B + a_3|1\rangle_A|0\rangle_B + a_4|1\rangle_A|1\rangle_B \quad (3)$$

where $a_i (i = 1, 2, 3, 4)$ are the probability amplitudes with $\sum_{i=1}^4 |a_i|^2 = 1$. We use the basis ($|1\rangle = |0\rangle_A|0\rangle_B$, $|2\rangle = |0\rangle_A|1\rangle_B$, $|3\rangle = |1\rangle_A|0\rangle_B$, $|4\rangle = |1\rangle_A|1\rangle_B$) to define the density matrix of the two qubit system. The equations of motion in terms density matrix elements can be obtained using the master equation 2 and with their solutions in the general case are given in the Appendix A.

3. Degree of entanglement

To study the effect of interaction among the two-bipartite on decoherence we have to investigate the dynamics of two-bipartite entanglement. In order to compare the degree of the entanglement restrained to quantum state, we will use two entanglement measures, i.e., logarithmic negativity and concurrence, to describe the degree of entanglement for any bipartite system. Both measures satisfy necessary conditions for being good measures of entanglement. The logarithmic negativity [40, 44] for two-bipartite system is defined by

$$\mathcal{N} = \log_2 \|\rho^{T_B}\|_1 \quad (4)$$

where ρ^{T_B} is the partial transpose of a state ρ in $d \otimes d'$ ($d \leq d'$) quantum system and $\|\cdot\|_1$ is the trace norm that can be read as

$$\|\rho^{T_B}\|_1 = 1 + 2 \left| \sum_i \mu_i \right| \quad (5)$$

where μ_i are the negative eigenvalues of ρ^{T_B} . For pure states, $\mathcal{N} = 0$ for unentangled states and $\mathcal{N} = 0$ for the maximally entangled state.

We will also consider another important measure of entanglement that is the concurrence [38, 45],

$$C(t) = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}) \quad (6)$$

where λ 's are the eigenvalues of the non-hermitian matrix $\rho(t)\tilde{\rho}(t)$ arranged in decreasing order of the magnitude. The matrix $\rho(t)$ is the density matrix for the two-bipartite and the matrix $\tilde{\rho}(t)$ is given by

$$\tilde{\rho}(t) = (\sigma_y^A \otimes \sigma_y^B) \rho^*(t) (\sigma_y^A \otimes \sigma_y^B) \quad (7)$$

where $\rho(t)^*$ is the complex conjugation of $\rho(t)$ and σ_y is the Pauli matrix given in quantum mechanics. The concurrence fluctuates between $C = 0$ for a separable state and $C = 1$ for a maximally entangled state. The two measures of entanglement are different for mixed quantum states.

Here we will consider some interesting initial entangled states for the two-bipartite which can be prepared and have potential applications in the quantum information processing tasks [1–6].

We will start by the investigation of the EPR-states which are concepts in quantum information science, a crucial part of quantum teleportation and represent the simplest possible examples of entanglement.

1. Assume that the initially entangled state of the field in two cavities to be in a NOON state given by

$$|\Psi\rangle_{AB}(0) = a_2|0\rangle_A|1\rangle_B + a_3|1\rangle_A|0\rangle_B \quad (8)$$

This kind of state can be generated by passing a two-level atom initially in the excited state through the two empty high-Q cavities. The interaction times of an atom with two cavities are chosen to be such that we have a $\pi/2$ pulse in the first cavity and a π pulse in the second cavity [46]. The initial logarithmic negativity and concurrence are given by

$$\mathcal{N}(0) = \log_2(1 + 2|\rho_{23}(0)|) = \log_2(1 + 2|a_2a_3|) \quad (9)$$

$$C(0) = 2|\rho_{23}(0)| = 2|a_2 a_3| \quad (10)$$

where the two quantities are related for this case by $\mathcal{N}(0) = \log_2(1 + C(0))$. It is evident, the solution of this equation gives $\mathcal{N}(0) = C(0) = 1$ which corresponds to the case $|a_2| = |a_3| = \frac{1}{\sqrt{2}}$. In (Figure A.2) the variation of the initial values of the logarithmic negativity and concurrence in terms of $|a_2|$ is presented where $|a_2|^2 + |a_3|^2 = 1$.

Using the solutions of Appendix A, it can be shown that the density matrix $\rho(t)$ can be read as

$$\rho(t) = \begin{pmatrix} \rho_{11}(t) & 0 & 0 & 0 \\ 0 & \rho_{22}(t) & \rho_{23}(t) & 0 \\ 0 & \rho_{23}^*(t) & \rho_{33}(t) & 0 \\ 0 & 0 & 0 & \rho_{44}(t) \end{pmatrix} \quad (11)$$

Then, we can calculate the negativity defined by (14) for the two-bipartite. We find

$$\begin{aligned} \mathcal{N}(t) = & \max\left(0, \right. \\ & \left. \log_2 \left[1 - \rho_{11}(t) - \rho_{44}(t) + \sqrt{[\rho_{11}(t) - \rho_{44}(t)]^2 + 4|\rho_{23}(t)|^2} \right] \right) \end{aligned} \quad (12)$$

while the concurrence defined by (6) is given by

$$C(t) = \max\left(0, 2 \left[|\rho_{23}(t)| - \sqrt{\rho_{11}(t)\rho_{44}(t)} \right] \right) \quad (13)$$

As they decay, they get entangled with the environment, slowly losing their coherence and purity over time.

For the case of vacuum reservoir, we can see from the solutions of the Appendix A that $\rho_{44}(t) = 0$. Then the relation between concurrence and logarithmic negativity can be given by

$$\mathcal{N}(t) = \max\left(0, \log_2 \left[1 - \rho_{11}(t) + \sqrt{\rho_{11}(t)^2 + C^2} \right] \right) \quad (14)$$

$$C = 2|\rho_{23}(t)| \quad (15)$$

where we can not observe in this case the ESD for any initial states.

2. Consider now the initially entangled two-bipartite to be in a another EPR-state given by

$$|\Psi\rangle_{AB}(0) = a_1|0\rangle_A|0\rangle_B + a_4|1\rangle_A|1\rangle_B \quad (16)$$

This kind of states can be prepared by swapping the vacuum and one-photon state in the cavity A of the state (8) discussed above [21, 47]. States like this have been realized in experiments with trapped ions [48, 49]. The initial logarithmic negativity and concurrence read as

$$\mathcal{N}(0) = \log_2(1 + 2|\rho_{14}(0)|) = \log_2(1 + 2|a_1a_4|) \quad (17)$$

$$C(0) = 2|\rho_{14}(0)| = 2|a_1a_4| \quad (18)$$

where the maximum value of $\mathcal{N}(0)$ and $C(0)$ are equal 1 for $|a_1| = |a_4| = \frac{1}{\sqrt{2}}$, where $|a_1|^2 + |a_4|^2 = 1$. The initial values of logarithmic negativity and concurrence have the same behavior (Figure A.2) in terms of $|a_1|$.

Exploiting the solutions of the Appendix A, the density matrix $\rho(t)$ can have the form

$$\rho(t) = \begin{pmatrix} \rho_{11}(t) & 0 & 0 & \rho_{14}(t) \\ 0 & \rho_{22}(t) & 0 & 0 \\ 0 & 0 & \rho_{33}(t) & 0 \\ \rho_{14}^*(t) & 0 & 0 & \rho_{44}(t) \end{pmatrix} \quad (19)$$

We can show that the logarithmic negativity defined by (5) and the concurrence defined by (6) for the two-bipartite in this case are given by

$$\mathcal{N}(t) = \max\left(0, \log_2 \left[1 - \rho_{22}(t) - \rho_{33}(t) + \sqrt{[\rho_{22}(t) - \rho_{33}(t)]^2 + 4|\rho_{14}(t)|^2} \right] \right) \quad (20)$$

and

$$C(t) = \max\left(0, 2 \left[|\rho_{14}(t)| - \sqrt{\rho_{22}(t)\rho_{33}(t)} \right] \right) \quad (21)$$

As is evident from the solutions in Appendix A, in this case $\rho_{22}(t) = \rho_{33}(t)$. Then the relation between concurrence and logarithmic negativity can be given by

$$\mathcal{N}(t) = \max\left(0, \log_2[1 + \tilde{C}(t)]\right) \quad (22)$$

$$C(t) = \max(0, \tilde{C}(t)) \quad (23)$$

where

$$\tilde{C}(t) = 2[|\rho_{14}(t)| - \rho_{22}(t)] \quad (24)$$

which makes clear that the logarithmic negativity is greater than the concurrence, when C decreases in terms of t , except for the initial value which is one for both of them for $|a_1| = |a_4| = \frac{1}{\sqrt{2}}$, and the value for $t \rightarrow \infty$ which is also zero.

4. Entanglement dynamics

We will explore the time evolution of entanglement of the previous entangled states of two-bipartite system exposed to either vacuum or thermal reservoirs. In figure A.3 we plot the logarithmic negativity and the concurrence for the first EPR state discussed previously in (i), for different initial states for vacuum reservoirs. Note that for vacuum reservoirs $\bar{n} = 0$, i.e. at zero temperature, no ESD is observed. $\mathcal{N}(t)$ and $C(t)$ monotonically decrease to zero as $t \rightarrow \infty$. On the other hand, the two measures give different values for entanglement in vacuum reservoirs. Generally, the logarithmic negativity takes smaller values than the concurrence [50], except for initial value for some initial states figure (A.3,b), this can be seen from the equations (14, 15) where the concurrence is proportional to the population $\rho_{23}(t)$ which monotonically decreases, while the logarithmic negativity depends on the population $\rho_{11}(t)$ which grows rapidly to reach a maximum value but the population $\rho_{44}(t)$ remains zero. In figure A.4 we present the two measures of entanglement for the thermal reservoirs in the case $|a_2| = |a_3| = \frac{1}{\sqrt{2}}$ and for different values of \bar{n} . Here the ESD is observed and as the temperature increases, the sudden death time decreases. On the other hand, we can observe the perfect correspondence between the logarithmic negativity and the concurrence for the thermal reservoirs while in the vacuum reservoirs the concurrence exceeds the logarithmic negativity which decays faster than the concurrence where the population $\rho_{44}(t)$ starts to manifest which restrains

the increasing population ρ_{11} . The sudden death time is identical with the two measures of entanglement. Indeed, for the 2 two-dimensional systems, logarithmic negativity and concurrence are real entanglement measures: they do not increase local operations and classical communication, and vanish if only if the state is separable.

In figure A.5 we plot the dynamical evolution of the entanglement when the second EPR state (ii) is considered, for $|a_1| = |a_4| = \frac{1}{\sqrt{2}}$ and for different values of \bar{n} . For this case no ESD in the vacuum reservoirs, but when $a_1 < a_4$ we can see a finite-time disentanglement, which means that the major contribution of this state is responsible for the ESD. Thus, we can conclude that locally equivalent pure states with the same entanglement perform very differently during the time evolution and simple local unitary operation acts on the initial state can give naissance to ESD. As the mean photon numbers in cavities increase, the ESD is always observed and the finite-time disentanglement persists as shown in figure A.6. Contrary to the first EPR state, the logarithmic negativity exceeds the concurrence and they are in good coincidence. Furthermore, the sudden death time is the same with the two measures of entanglement.

On the other hand, it is obvious that the two measures of entanglement are very closed. The sudden death time is the same in the two measures of entanglement, and the logarithmic negativity predicts the same behavior of the entanglement as the concurrence. When the average thermal photon number is different from zero, we can see that the ESD always occurs whatever the initial entangled states are and no matter how the nonzero average thermal photon number is. Furthermore, the two proposed measures of entanglement coincide and give a good prediction of the sudden death entanglement of the two-bipartite. This is consistent with the findings in [15, 16]

5. Conclusion

The delicate aspect of entanglement has been subjected to many quantitative studies and has guided to interesting results. Understanding the physical meaning of entanglement measures continues, however, a major defy. In this work we have shown that the scheme we propose here allows direct measures of entanglement: logarithmic negativity and concurrence of a two-bipartite cavity system. The sudden death time depends on the number of photons in the cavities and the temperature of the reservoirs. It increases with increasing the number of photons in the cavities and decreases with increasing

temperature of the reservoirs. The proposed scheme only involves system-reservoir interactions corresponding to the decay of the radiation field inside a cavity (field damping). This operation is based on the elementary exchange of energy between system and reservoir that is thus assumed to consist of the simultaneous creation of a quantum excitation of the system with annihilation of a quantum in one mode of the reservoir, or reverse process. These operations have been demonstrated experimentally [51] and therefore our proposed scheme can be realized within the present cavity-QED technologies. As a future perspective it would be interesting to study the effect of other type of environment like a squeezed reservoirs. Further, It would be motivating to enlarge our point of view to the two-bipartite entanglement of high dimension for the study disentanglement and loss of decoherence, where the Wootter's criterion is not applicable and we can use logarithmic negativity which is necessary and sufficient condition for 2×3 and 3×2 systems.

Appendix A. Equations of motion of the density matrix elements and their solutions for vacuum reservoir

The equation of motion of density matrix elements for the general state (Eq. 3) are given by

$$\begin{aligned}
\dot{\rho}_{11}(t) &= (-\bar{n}_A \gamma^{(A)} - \bar{n}_B \gamma^{(B)}) \rho_{11}(t) + (\bar{n}_A + 1) \gamma^{(A)} \rho_{33}(t) + (\bar{n}_B + 1) \gamma^{(B)} \rho_{22}(t) \\
\dot{\rho}_{12}(t) &= -\left[\frac{1}{2}(2\bar{n}_B + 1) \gamma^{(B)} + \bar{n}_A \gamma^{(A)}\right] \rho_{12}(t) + (\bar{n}_A + 1) \gamma^{(A)} \rho_{34}(t) \\
\dot{\rho}_{13}(t) &= -\left[\frac{1}{2}(2\bar{n}_A + 1) \gamma^{(A)} + \bar{n}_B \gamma^{(B)}\right] \rho_{13}(t) + (\bar{n}_B + 1) \gamma^{(B)} \rho_{24}(t) \\
\dot{\rho}_{14}(t) &= -\left[(\bar{n}_A + \frac{1}{2}) \gamma^{(A)} + \gamma^{(B)}(\bar{n}_B + \frac{1}{2})\right] \rho_{14}(t) \\
\dot{\rho}_{21}(t) &= -\left[\left(\frac{1}{2}(2\bar{n}_B + 1)\right) \gamma^{(B)} + \bar{n}_A \gamma^{(A)}\right] \rho_{21}(t) + (\bar{n}_A + 1) \gamma^{(A)} \rho_{43}(t) \\
\dot{\rho}_{22}(t) &= -[\bar{n}_A \gamma^{(A)} + (\bar{n}_B + 1) \gamma^{(B)}] \rho_{22}(t) + (\bar{n}_A + 1) \gamma^{(A)} \rho_{44}(t) + \bar{n}_B \gamma^{(B)} \rho_{11}(t) \\
\dot{\rho}_{23}(t) &= -\left[(\bar{n}_A + \frac{1}{2}) \gamma^{(A)} + (\bar{n}_B + \frac{1}{2}) \gamma^{(B)}\right] \rho_{23}(t) \\
\dot{\rho}_{24}(t) &= -\left[(\bar{n}_B + 1) \gamma^{(B)} + (\bar{n}_A + \frac{1}{2}) \gamma^{(A)}\right] \rho_{24}(t) + \bar{n}_B \gamma^{(B)} \rho_{13}(t) \\
\dot{\rho}_{31}(t) &= -\left[\frac{1}{2}(2\bar{n}_A + 1) \gamma^{(A)} + \bar{n}_B \gamma^{(B)}\right] \rho_{31}(t) + (\bar{n}_B + 1) \gamma^{(B)} \rho_{42}(t) \\
\dot{\rho}_{32}(t) &= -\left[(\bar{n}_A + \frac{1}{2}) \gamma^{(A)} + (\bar{n}_B + \frac{1}{2}) \gamma^{(B)}\right] \rho_{32}(t)
\end{aligned}$$

$$\begin{aligned}
\dot{\rho}_{33}(t) &= -[(\bar{n}_A + 1)\gamma^{(A)} + \bar{n}_B\gamma^{(B)}]\rho_{33}(t) + \bar{n}_A\gamma^{(A)}\rho_{11}(t) + (\bar{n}_B + 1)\gamma^{(B)}\rho_{44}(t) \\
\dot{\rho}_{34}(t) &= -[(\bar{n}_A + 1)\gamma^{(A)} + (\bar{n}_B + \frac{1}{2})\gamma^{(B)}]\rho_{34}(t) + \bar{n}_A\gamma^{(A)}\rho_{12}(t) \\
\dot{\rho}_{41}(t) &= -[(\bar{n}_A + \frac{1}{2})\gamma^{(A)} + (\bar{n}_B + \frac{1}{2})\gamma^{(B)}]\rho_{41}(t) \\
\dot{\rho}_{42}(t) &= -[(\bar{n}_A + \frac{1}{2})\gamma^{(A)} + (\bar{n}_B + 1)\gamma^{(B)}]\rho_{42}(t) + \bar{n}_B\gamma^{(B)}\rho_{31}(t) \\
\dot{\rho}_{43}(t) &= -[(\bar{n}_A + 1)\gamma^{(A)} + (\bar{n}_B + \frac{1}{2})\gamma^{(B)}]\rho_{43}(t) + \bar{n}_A\gamma^{(A)}\rho_{21}(t) \\
\dot{\rho}_{44}(t) &= -[(\bar{n}_A + 1)\gamma^{(A)} + (\bar{n}_B + 1)\gamma^{(B)}]\rho_{44}(t) + \bar{n}_A\gamma^{(A)}\rho_{22}(t) + \bar{n}_B\gamma^{(B)}\rho_{33}(t)
\end{aligned}$$

For the sake of simplicity, we assume that the cavities are identical $\gamma^{(A)} = \gamma^{(B)} = \gamma$ and $\bar{n}_A = \bar{n}_B = \bar{n}$. On solving these equations we get the time evolution of the density elements matrix

$$\begin{aligned}
\rho_{11}(t) &= \frac{1}{4a^2}\{[-2 + 2a(2\rho_{11}(0) + \rho_{22}(0) + \rho_{33}(0) - 1)](1 + a)e^{-\eta t} \\
&\quad + [1 + a^2(1 - 2\rho_{33}(0) - 2\rho_{22}(0)) \\
&\quad - 2a(2\rho_{11}(0) + \rho_{22}(0) + \rho_{33}(0) - 1)]e^{-2\eta t} \\
&\quad + (1 + a)^2\} \\
\rho_{22}(t) &= \frac{1}{4a^2}\{[-1 + a^2(2\rho_{22}(0) + 2\rho_{33}(0) - 1) \\
&\quad + 2a(2\rho_{11}(0) + \rho_{22}(0) + \rho_{33}(0) - 1)]e^{-2\eta t} \\
&\quad + [2 + 2a^2(\rho_{22}(0) - \rho_{33}(0)) - 2a(2\rho_{11}(0) + \rho_{22}(0) + \rho_{33}(0) - 1)]e^{-\eta t} \\
&\quad + (a^2 - 1)\} \\
\rho_{33}(t) &= \frac{1}{4a^2}\{[-1 + a^2(2\rho_{22}(0) + 2\rho_{33}(0) - 1) \\
&\quad + 2a(2\rho_{11}(0) + \rho_{22}(0) + \rho_{33}(0) - 1)]e^{-2\eta t} \\
&\quad + [2 + 2a^2(\rho_{33}(0) - \rho_{22}(0)) - 2a(2\rho_{11}(0) + \rho_{22}(0) + \rho_{33}(0) - 1)]e^{-\eta t} \\
&\quad + (a^2 - 1)\} \\
\rho_{44}(t) &= 1 - \rho_{11} - \rho_{22} - \rho_{33} \\
\rho_{12}(t) &= \frac{1}{2a}\{[a(\rho_{12}(0) - \rho_{34}(0)) - \rho_{34}(0) - \rho_{12}(0)]e^{-\frac{3}{2}\eta t} \\
&\quad + (\rho_{34}(0) + \rho_{12}(0))(1 + a)e^{-\frac{1}{2}\eta t}\} \\
\rho_{13}(t) &= \frac{1}{2a}\{[a(-\rho_{24}(0) + \rho_{13}(0)) - \rho_{24}(0) - \rho_{13}(0)]e^{-\frac{3}{2}\eta t} \\
&\quad + (\rho_{24}(0) + \rho_{13}(0))(1 + a)e^{-\frac{1}{2}\eta t}\}
\end{aligned}$$

$$\begin{aligned}
\rho_{14}(t) &= \rho_{14}(0)e^{-\eta t} \\
\rho_{23}(t) &= \rho_{23}(0)e^{-\eta t} \\
\rho_{24}(t) &= \frac{1}{2a}\{a(\rho_{24}(0) - \rho_{13}(0)) + \rho_{24}(0) + \rho_{13}(0)\}e^{-\frac{3}{2}\eta t} \\
&\quad + (\rho_{24}(0) + \rho_{13}(0))(-1 + a)e^{-\frac{1}{2}\eta t} \\
\rho_{34}(t) &= \frac{1}{2a}\{a(\rho_{34}(0) - \rho_{12}(0)) + \rho_{34}(0) + \rho_{12}(0)\}e^{-\frac{3}{2}\eta t} \\
&\quad + (\rho_{34}(0) + \rho_{12}(0))(-1 + a)e^{-\frac{1}{2}\eta t}
\end{aligned}$$

and $\rho_{21}(t) = \rho_{12}(t)$, $\rho_{31}(t) = \rho_{13}(t)$, $\rho_{32}(t) = \rho_{23}(t)$, $\rho_{41}(t) = \rho_{14}(t)$, $\rho_{42}(t) = \rho_{24}(t)$, $\rho_{43}(t) = \rho_{34}(t)$, where $a = 2\overline{n} + 1$ and $\eta = a\gamma$.

References

- [1] C. H. Bennett ,G. Brassard ,C. Crepeau ,R. Jozsa ,A. Peres and W. K. Wttersoo, *Phys.Rev.Lett.* **70**,1895(1993) .
- [2] D. Bouwmeester , J W Pan , K Mattle , M Eibl , H. Weinfurter and A. Zeilinger, *Nature(London)* **390**, 575(1997).
- [3] D.Boschi , S. Branca ,F. De Martini ,L. Hardy and S. Popescu, *Phy.Rev.Lett.* **80** 1998, 1121(1998).
- [4] A. Furusawa , J. L. Sorensen ,S. L. Braunstein ,C. A. Fuchs , H.J. Kimble and E.S. Polzik, *Science* **282**, 706(1998) .
- [5] C. H. Bennett and S.J. Wiesner, *Phys. Rev. Lett.* **69** 2881(1992) .
- [6] A. K. Ekert , *Phy.Rev.Lett.* **67** 661(1991).
- [7] H. J. Briegel and R. Raussendorf, *Phys. Rev. Lett.* **86**, 910; R. Raussendorf and H. J. Briegel, *Phys. Rev. Lett.* **86** 5188(2001).
- [8] S. J. van Enk , J. I. Cirac and P. Zoller , *Phys. Rev. Lett.* **79** 5178(1997).
- [9] A. Rauschenbeutel , G. Nogues ,S. Osnaghi ,P. Bertet ,M. Brune ,J.M. Raimond and S. Haroche, *Phys. Rev. Lett.* **83** 5166(1999) .
- [10] M.C. de Oliveira and W.J. Munro, *Phys. Rev. A* **61** 042309(2000).
- [11] E. Jane ,M.B. Plenio and D. Jonathan , *Phys. Rev. A* **65** 050302(2002).
- [12] S. B. Zheng and G. C. Guo , *Phys. Rev. A* **63** 044302(2001).
- [13] F. Mintert , M. Kus and A. Buchleitner, /quant-ph/0403063v1,(2004)
- [14] D. E. Browne and M. B. Plenio , *Phys. Rev. A* **67** 012325(2003).
- [15] M. Ikram , F. L. Li and M. S. Zubairy, *Phys. Rev. A* **75** 062336(2007).
- [16] R.Tahira , M. Ikram , T. Azim , and M.S. Zubairy, *J. Phys. B: At. Mol. Opt. Phys.* **41** 205501(2008).
- [17] S. B. Li and J. B. Xu , *J.Optics B.* **6**(9) 373(2004).

- [18] T. Yu and J. H. Eberly , *Phys. Rev. Lett.* **93** 140404(2004). T. Yu and J. H. Eberly , *Phys. Rev. Lett.* **97** 140403(2006);T. Yu and J. H. Eberly , *Opt. Commun.* **264** 393(2006);M. Yonac , T. Yu and J H Eberly , *J. Phys. B: At. Mol. Opt. Phys.* **39** S621(2006); J. H. Eberly and T Yu, *Science* **316** 555(2007).
- [19] M. P. Almeida ,F. de Melo ,M. Hor-Meyll ,A. Salles , S. P. Walborn , P. H. Souto Ribeiro ,L. Davidovich, *Science* **316**, 579(2007).
- [20] J. Laurat , K. S. Choi , H. Deng , C. W. Chou , and H. J. Kimble , *Phys. Rev. Lett.* **99** 180504(2007) .
- [21] M. Frana Santos , P. Milman , L. Davidovich and N. Zagury , *Phys. Rev. A* **73** 040305(R)(2006).
- [22] P. Marek , J. Lee and M. S. Kim, *Phys. Rev. A* **77** 032302(2008).
- [23] Y.S. Gong Yan-Xiao,Zhang,Y.L. Dong, X.L. Niu, Y. F. Huang and G. C. Guo , *Phys. Rev. A* **78** 042103(2008).
- [24] D. Tolkunov , V. Privman and P. K. Aravind, *Phys. Rev. A* **71** 060308(R)(2005).
- [25] C. H. Chou, T. Yu and B. L. Hu , *Phys. Rev. E* **77** 011112(2008).
- [26] C. E. Lopez ,G. Romero ,F. Lastra ,E. Solano and J. C. Retamal, *Phys. Rev. Lett.* **101** 080503(2008).
- [27] C. Cormick and J. P. Paz , *Phys. Rev. A* **78** 012357(2008).
- [28] C. Y. Lai , J. T. Hung ,C. Y. Mou and P. Chen , *Phys. Rev. B* **77** 205419(2008).
- [29] A. Abliz ,H. J. Gao , X. C. Xie , Y. S. Wu and W. M. Liu , *Phys. Rev. A* **74** 052105(2006).
- [30] Z. Ficek and R. Tanas, *Phys. Rev. A* **74** 024304(2006).
- [31] Z.Ficek , and R. Tanas, *Phys. Rev. A* **77** 054301(2008).
- [32] S. Das and G. S. Agarwal , quant-ph/0905.3399v1(2009).

- [33] A.Salles, F. de Melo ,J. C. Retamal, R. L. de Matos Filho and N. Zagury, *Phys. Rev. A* **74** 060303(R)(2006).
- [34] O. Ghne , P. Hyllus , D. Bruss , A. Ekert , M. Lewenstein ,C Macchiavello and A. Sanpera , *Phys. Rev. A* **66** 062305(2002).
- [35] P. Horodecki and A. Ekert *Phys. Rev. Lett.* **89** 127902(2002).
- [36] P. Horodecki, *Phys. Rev. Lett.* **90** 167901(2003).
- [37] J. M. Cai , Z. W. Zhou , and G. C. Guo , *Phys. Rev. A* **73** 024301(2006).
- [38] W. K. Wootters, *Phys. Rev. Lett.* **80** 2245(1998).
- [39] S. P. Walborn, P. H. Souto Ribeiro , L. Davidovich , F. Mintert and A. Buchleitner , *Nature(London)* **440** 1022(2006).
- [40] G. Vidal and R. F. Werner , *Phys. Rev. A* **65** 032314(2002).
- [41] A. Peres A, *Phys. Rev. Lett.* **76** 1413(1996).
- [42] M. O. Scully and M. S. Zubairy , *Quantum Optics*, Cambridge University Press, (1997)
- [43] M. S. Zubairy, G. S. Agarwal and M O Scully , *Phys. Rev. A* **70** 012316(2004).
- [44] S. Lee , D. P. Chi , S. D. Oh and J. Kim, *arXiv:quant-ph/0310027v1*,(2003).
- [45] S. P. Walborn , P. S. Ribeiro , L. Davidovich , F. Mintert and A. Buchleitner, *Nature (London)* **440** 1022(2006)
- [46] L. Davidovich, N. Zagury, M. Brune, J.M. Raimond and S. Haroche, *Phys.Rev.A* **50**, R895 (1994).
- [47] M. Ikram, S. Zhu, and M. S. Zubairy, *Phys. Rev. A* **62**, 022307 (2000)
- [48] D. Leibfried et al., *Nature*,**422**,412(2003).
- [49] C. F. Roos, G. P. T. Lancaster, M. Riebe, H. H?ffner, W. H?nsel, S. Gulde, C. Becher, J. Eschner, F. Schmidt-Kaler and R. Blatt, *Phys. Rev. Lett.* **92**, 220402 (2004).

- [50] R. Tanas and Z.Ficek, *J. Opt. B* **6** S90-S97(2004).
- [51] P. Bertet , S. Osnaghi ,A. Rauschenbeutel ,G. Nogues, A. Auffeves ,M. Brune , J. M. Raimond and S. Haroche, *Nature(London)* **411** 166(2001).

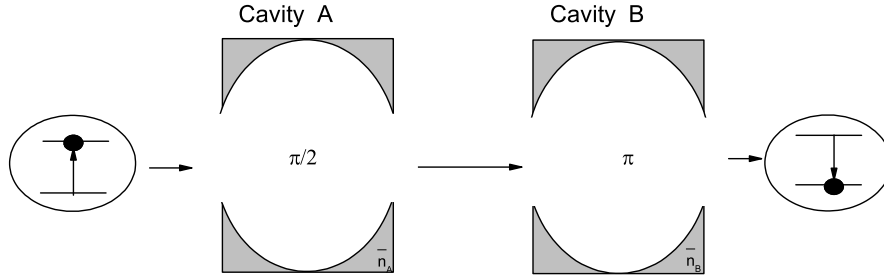


Figure A.1: The scheme for single-particle interference. A two-level atom prepared in its excited state passes successively through cavity A, a field damping region and cavity B. At the end the two-level atom will be in its ground state.

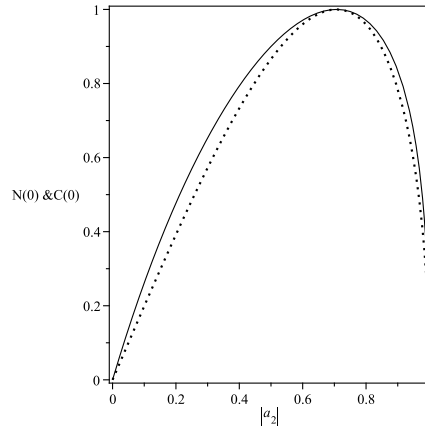


Figure A.2: Initial value of logarithmic negativity (line) and Concurrence (dot) in terms of $|a_2|$ where $|a_2|^2 + |a_3|^2 = 1$

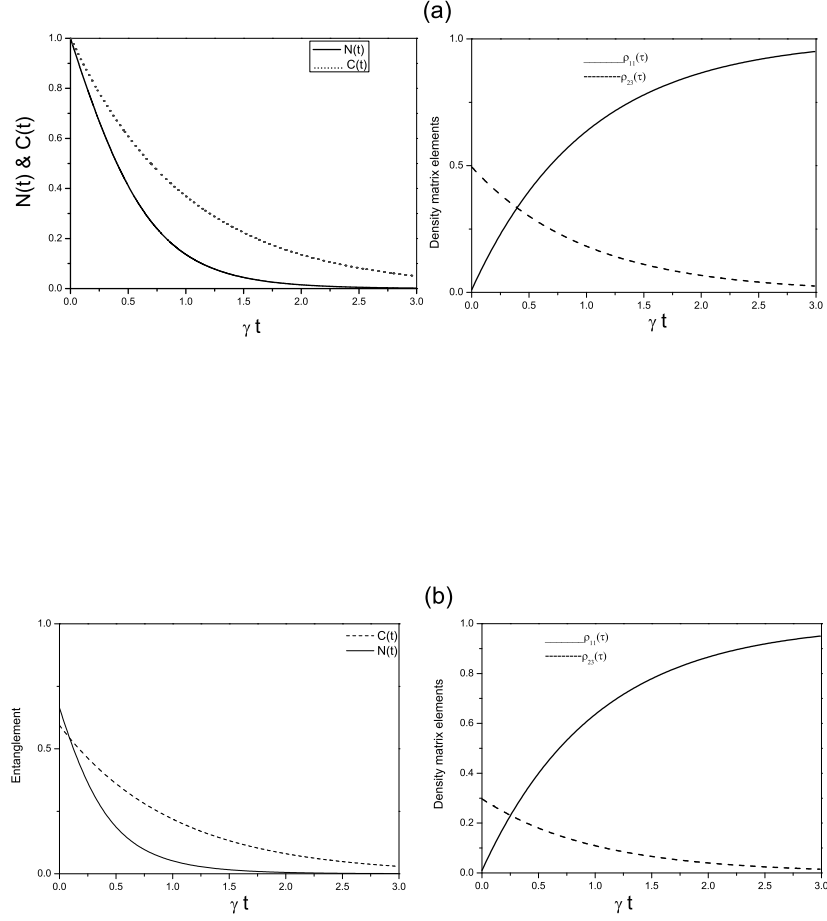


Figure A.3: Entanglement dynamics of the first EPR state for different initial probability for vacuum reservoirs. (a) $|a_2| = \frac{1}{\sqrt{10}}$, (b) $|a_2| = \frac{1}{\sqrt{2}}$.

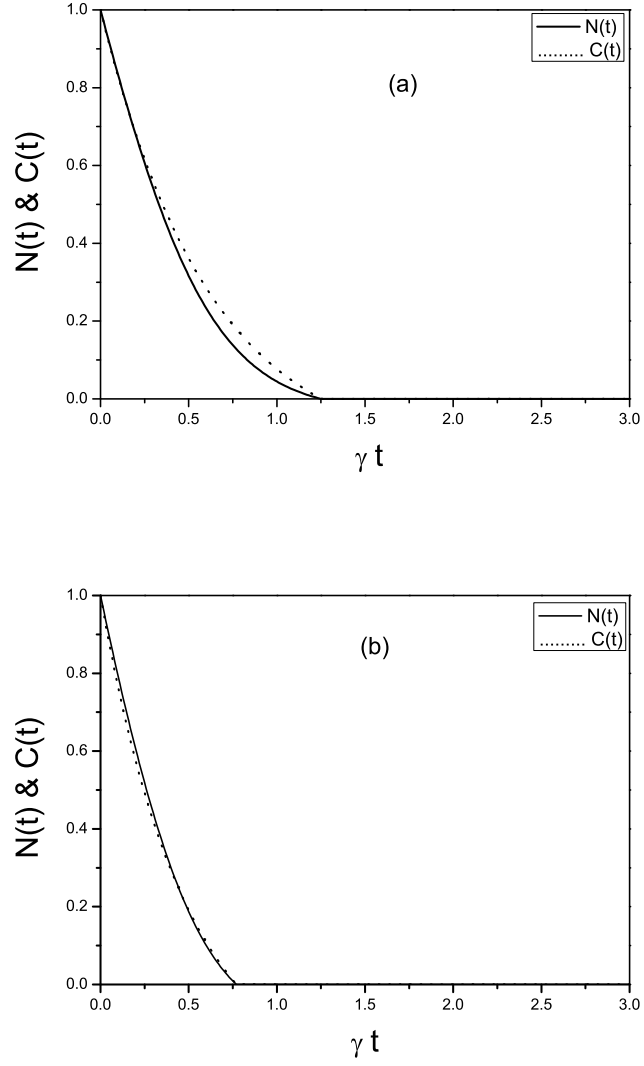


Figure A.4: Entanglement dynamics of the first EPR state at different mean photon numbers of reservoirs in the case $|a_2| = |a_3| = \frac{1}{\sqrt{2}}$. (a) $\bar{n} = 0.1$, (b) $\bar{n} = 0.25$.

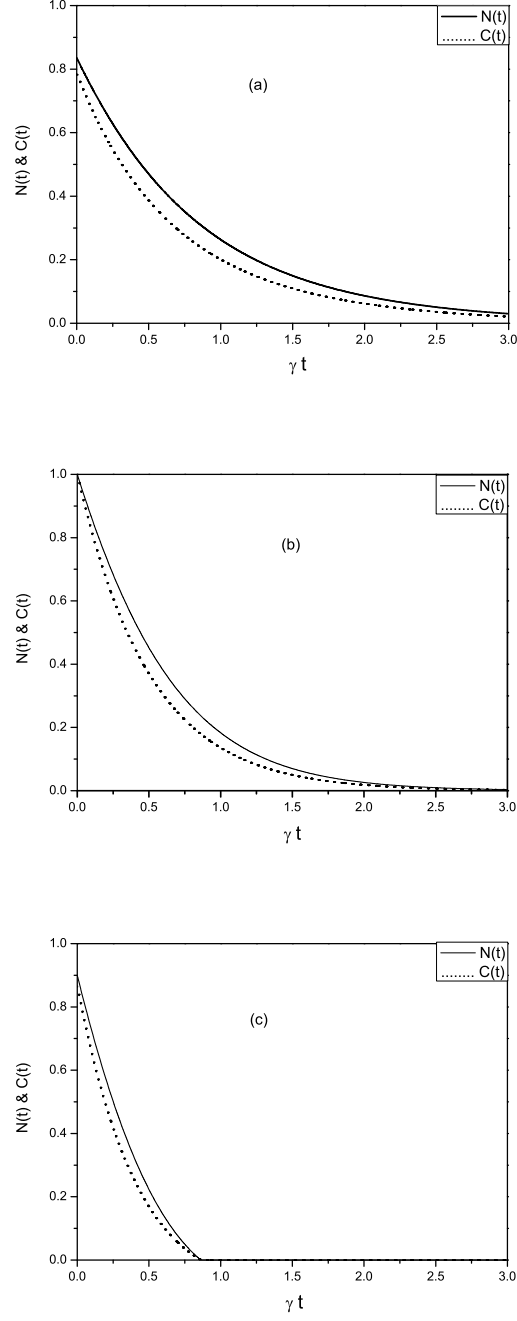


Figure A.5: Entanglement dynamics of the second EPR state at different initial probability for vacuum reservoirs. (a) $|a_1| = 0.9$, (b) $|a_1| = \frac{1}{\sqrt{2}}$, (c) $|a_1| = 0.5$.

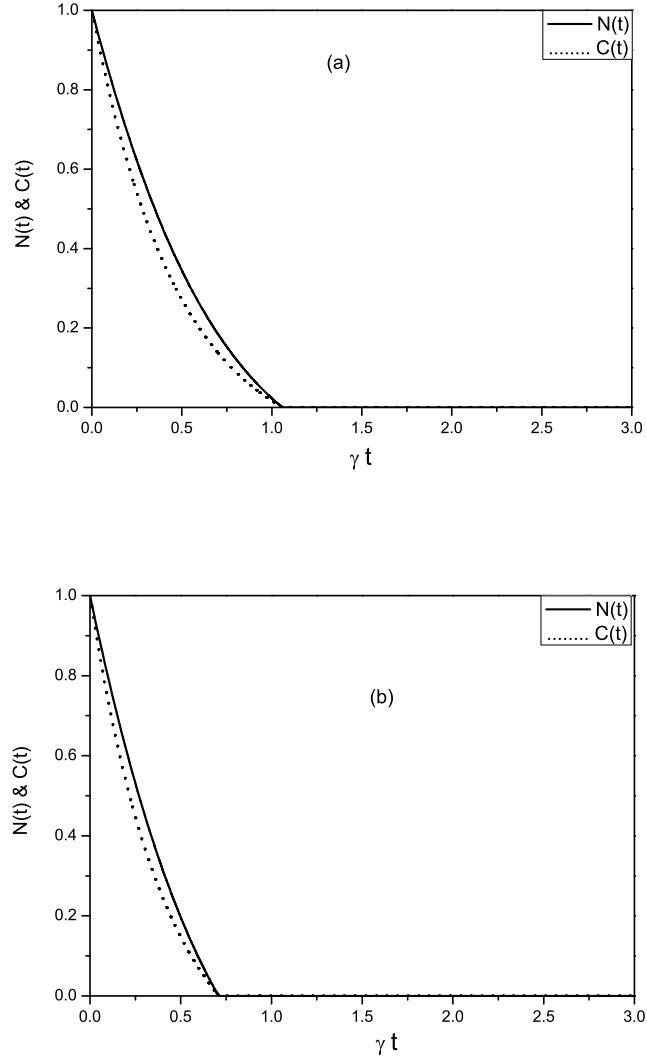


Figure A.6: Entanglement dynamics of the second EPR state at different mean photon numbers of reservoirs in the case $|a_1| = |a_4| = \frac{1}{\sqrt{2}}$ (a) $\bar{n} = 0.1$, (b) $\bar{n} = 0.25$.